the discounted cash flow internal rate of return

relevant to CAT Scheme Papers 4 and 10

capital investment appraisal

■ Sessions 18 and 19 of the *Study Guide* for CAT Paper 4 are concerned with the use of discounted cash flow (DCF) methods in the appraisal of capital investment projects. These sessions on DCF cover the mathematics of discounting and how it is used to evaluate the worth of capital investment projects via an analysis of cash flows. The fact that these are the last sections in the *Study Guide*, and are invariably covered last in study materials, may account for the poor performance by the majority of candidates when tackling questions on this topic.

My article in the April 2006 issue of *student accountant* focused on the identification of relevant cash flows in an investment project, and on the discounting of those cash flows in the calculation of the net present value (NPV) of an investment. In that article, I highlighted the common errors made by candidates in these aspects of capital investment appraisal, using Question 1 from Section B of the December 2005 Paper 4 exam as illustration. Candidates are also frequently unclear as to how and when to use the internal rate of return (IRR) formula in capital investment appraisal, and this is the subject of this second article.

Understanding how the NPV of an investment changes as the discount rate is varied is a good start. A useful second step is an understanding of how linear interpolation and/or extrapolation can be used in the approximation of the IRR percentage return. Part (c) of Question 1 from Section B of the June 2007 Paper 4 exam (set out below), and my comments on candidate performance, will be used in this article to highlight and explain these aspects.

QUESTION

A capital investment project has estimated net cash inflows of $60,000 per annum for six years. Discounting the net cash flows at 10% and 20% per annum, the present values of the inflows are:

Annual discount rate	Present value of inflows
10%	$261,300
20%	$199,600

The initial investment amount is $224,000.

Required:

i   Plot the NPV of the project, at discount rates of 10% and 20% per annum, on graph paper.
ii Indicate, on the graph, an estimate of the IRR of the project.

NPV AND DISCOUNT RATE

To answer Part (i), the NPV of the investment at 10% and at 20% per annum must first be calculated. The NPV of the cash flows is the total present value of the cash inflows (already given in the question), less the initial investment amount. Thus:

☐ at 10% NPV = $37,300 ($261,300 - $224,000)
☐ at 20% NPV = ($24,400) ($199,600 - $224,000)

The NPV of any investment that has a conventional stream of cash flows (ie investment outflow followed by a sequence of cash inflows) will decline as the discount rate increases. There is not, however, a linear relationship between discount rate and NPV because of the way that discounting works. This can be seen more clearly if additional discount factors are used to calculate further NPVs which can then be plotted on a graph (although this is not required to answer this question).
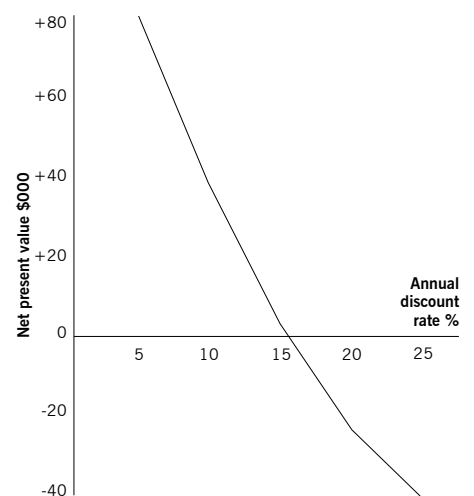
If discount rates of 5%, 15%, and 25% are also used, the further NPVs are:

☐ at 5% NPV = $80,560 [($60,000 x 5.076*) - $224,000]
☐ at 15% NPV = $3,040 [($60,000 x 3.784*) - $224,000]
☐ at 25% NPV = ($46,940) [($60,000 x 2.951*) - $224,000]

* Annuity rates (ie cumulative discount factors) over six years.

The five NPVs (at 5%, 10%, 15%, 20% and 25%) are plotted on **Graph 1**.
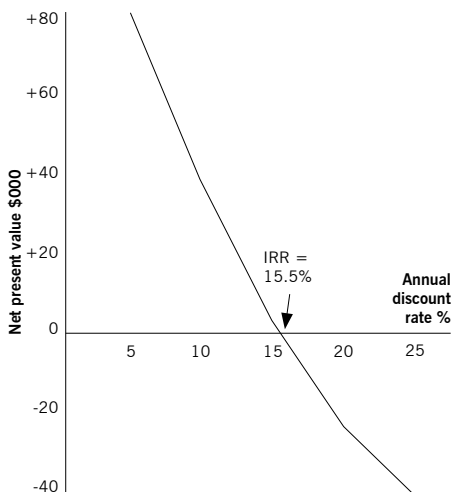
GRAPH 1: THE NPV CURVE

The plotted values, when joined up, demonstrate that the NPV declines in a gentle curve as the discount rate is increased. For each percentage increase in the discount rate the NPV declines by a smaller and smaller absolute amount. Candidates were not required to draw a line on the graph in order to answer Part (i) of the June 2007 question, but they did need to do this for Part (ii).

THE INTERNAL RATE OF RETURN (IRR)
The IRR method of DCF involves finding the percentage rate which, when used to discount the cash flows expected from an investment, will produce an NPV of zero (ie where the total present value of the sequence of cash inflows is equal to the present value of the cash amount invested).

Looking at the NPV graph (repeated as **Graph 2**), the IRR is therefore the point where the NPV line crosses the horizontal axis (annual discount rate %). This is at a discount rate of 15.5% (ie the NPV is zero at this rate of discount).

GRAPH 2: INTERNAL RATE OF RETURN OF 15.5%

EXAMINER'S REPORT
In my examiner's report, I noted that many candidates failed to attempt Part (c) of Question 1 in the June 2007 exam, and that very few of the candidates who did attempt it presented good answers. Most candidates failed to deduct the capital investment amount of $224,000 from the present value of the cash inflows to arrive at the NPVs to plot on the graph, thus also making identification of the IRR impossible. Scaling and labelling of the graph was often poor, and a common error was to prepare a graph with time (years) on the horizontal axis, rather than annual discount rate percentage.

A significant number of candidates failed to appreciate that present values for the cash inflows had been provided in the question. As a result, they wasted valuable time calculating discount rates at 10% and 20% and then applying these rates to the cash flows to get the present values already provided. This demonstrated a lack of understanding or failure to read the question carefully. A significant number of candidates also wasted time attempting a calculation of the IRR; they should have been able to indicate this on the graph using their plotted data, and therefore without calculations. In any case, very few candidates made any attempt at indicating the IRR on their graph.

CALCULATION OF THE IRR
Some exam questions on the topic of capital investment project appraisal require the calculation of the IRR % and candidates for Paper 4 have demonstrated that they struggle with this. In Part (b) of Question 4 from Section B of the June 2004 exam, candidates were required to calculate the IRR of a project from a given schedule of cash flows. In order to reduce the number of calculations that were required, candidates were given the NPV of the project when cash flows were discounted at a rate of 20% per annum. My examiner's report commented as follows:

Many candidates made a reasonable start by doing some discounting, at 10% and/or 14%, to get one or more further NPVs (although some calculated the NPV at 20% even though it was already provided in the question).

However, candidates often failed to carry it through either because they used an incorrect formula for IRR estimation or

> Candidates need to have an appreciation of the effect that the discounting process has on the NPV of an investment project at varying discount rates. They then need to be able to calculate an approximate IRR percentage return, without unnecessarily lengthy workings.

because they were unable to apply the formula correctly. Partly as a consequence, many candidates calculated the IRR to be greater than 20%. This was always without comment, despite the fact that they were informed in the question that the NPV was negative when the cash flows were discounted at 20%. It should, therefore, have been clear to candidates that the IRR had to be less than 20%. In addition, one of the reasons for the calculation of a high IRR was that candidates frequently simply discounted the positive cash inflows and failed to deduct the negative original investment. Candidates often estimated the IRR in this situation (ie estimated the discount rate at which the NPV would be zero) by using an extrapolation formula. It would actually be impossible to discount the cash flows to a zero NPV in such a situation because only positive cash flows were being included in the analysis. This impossibility was not recognised by any of the candidates.

INTERPOLATION AND EXTRAPOLATION FORMULA
As has been noted from **Graph 1**, there is no linear relationship between discount rate and NPV. As the discount rate is increased the NPV falls but, as indicated by the shape of the curve on the graph, the NPV reduces by an increasingly smaller absolute amount in response to successive changes of a certain amount in the discount rate.

The IRR can be found by repeated trial and error calculations but this process can be shortened, and still produce results within an acceptable range of values, by actually

assuming a linear relationship between NPV and discount rate to approximate the IRR. The key question is, when can such an assumption be made? We will return to this question later. First, the IRR approximation formula.

Any two NPVs can be used in the calculation of an approximate IRR for a capital investment project. If the two points used lie either side of the IRR (ie one discount rate lower than the IRR with a positive NPV, and one higher with a negative NPV) then the process of approximation is called interpolation (the calculation of an intermediate term). However, extrapolation (the calculation of a term outside the known range) is equally valid, using either two positive NPVs (and associated discount rates) or two negative NPVs.

The general formula for the estimation of the IRR of a capital investment project by interpolation or extrapolation is:

Approximate IRR = lower discount rate ±* [change in discount rate (%) between the two points x (NPV at lower discount rate ÷ change in NPV between the two points)]

* This will be + if the lower discount rate has a positive NPV. This will be - if the lower discount rate has a negative NPV

If interpolating, the factor (NPV at base point ÷ change in NPV between the two points used) < 1.

If extrapolating, the factor (NPV at base point ÷ change in NPV between the two points used) > 1.

Interpolation will always overstate the true return and thus should lead, if anything, to a rounding downwards. Extrapolation will always understate the true return and thus should lead, if anything, to a rounding upwards.
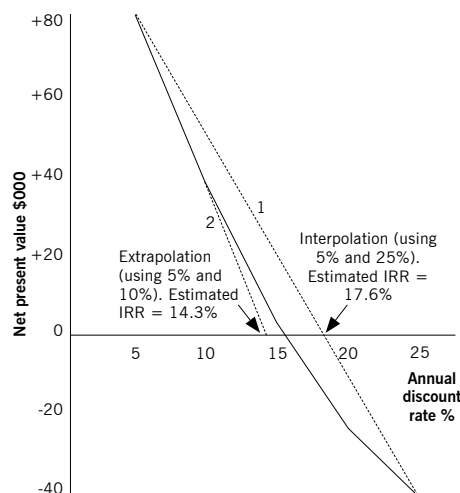
EXAMPLES OF INTERPOLATION AND EXTRAPOLATION
Returning to Part (c) of the June 2007 Paper 4 question used earlier, interpolation (estimating within the range) could be done using the NPVs at 5% and at 25% (one positive and one negative NPV). Using the above formula:

Approximate IRR = 5% + [(25 - 5%) x ($80,560/$80,560 +* 46,940)] = 17.6%

* In interpolation, the two figures are always added because both a positive value and a negative value are involved.

The approximate IRR of 17.6% is the value shown on broken line 1 on **Graph 3**, ie a linear relationship is assumed between values at 5% and 25%.

GRAPH 3: APPROXIMATE INTERNAL RATE OF RETURN

The interpolation seems an unreasonable approximation because the estimate of 17.6% is a significant overstatement when compared with our previous figure of 15.5%. This is because of the two points used. Taking the two points closest to the IRR (at 15% and 20%) the interpolation calculation confirms the previous reading from the graph as follows:

Approximate IRR = 15% + [(20 - 15%) x ($3,040/$3,040 + 24,400)] = 15.6%

Extrapolation (estimating outside of the range) could be done using, for example, the NPVs at 5% and 10% (both positive):

Approximate IRR = 5% + [(10 - 5%) x ($80,560/$80,560 -* 37,300)] = 14.3%

* In extrapolation, one NPV is always deducted from the other because either two positive values or two negative values are involved.

The approximate IRR of 14.3% is the value shown on broken line 2 of **Graph 3**, ie a linear relationship is assumed between the values at 5% and 10%, then extended to the horizontal axis. A closer approximation would be obtained by extrapolating using values closer to the IRR, for example at 10% and 15% discount rates. So:

Approximate IRR = 10% + [(15 - 10%) x ($37,300/$37,300 - 3,040)] = 15.4%

IRR APPROXIMATION RULE
As a general rule, two values can reasonably be used in interpolation or extrapolation as long as either each NPV is less than 20% of the undiscounted value* of the investment project, or one of the NPVs is within 10% of the undiscounted value (which could be the other value used for the interpolation/extrapolation). This should produce a sufficiently accurate approximation of the IRR percentage, especially if sensibly rounded down, if interpolation, or up if extrapolation, to the nearest whole percentage.

* In our example, the undiscounted value is $136,000, ie [($60,000 × 6) - $224,000].

CONCLUSION
Candidates for Paper 4 need to have an appreciation of the effect that the discounting process has on the NPV of an investment project at varying discount rates. They then need to be able to calculate an approximate IRR percentage return, without unnecessarily lengthy workings, by knowing the interpolation/extrapolation formula and by using it correctly and sensibly. It is hoped that this article will help candidates in these aspects of DCF capital investment appraisal. ■