Professional Level – Options Module

Advanced Financial Management

Friday 8 June 2018

P4 ACCA

Time allowed: 3 hours 15 minutes

This question paper is divided into two sections:

Section A – This ONE question is compulsory and MUST be attempted Section B – TWO questions ONLY to be attempted

Formulae and tables are on pages 9–13.

Do NOT open this question paper until instructed by the supervisor.

This question paper must not be removed from the examination hall.



The Association of Chartered Certified Accountants

Think Ahead ACCA



Section A – This question is compulsory and MUST be attempted

1 Ruva Co, a large listed engineering company, is reviewing its investment and financing strategy for the coming five years. Part of this strategy involves considering future investment projects and the company's finance team has submitted the following five projects for further consideration. The finance team feels that some past projects, although giving a small negative net present value (NPV) and therefore being rejected, might have been considered further. Therefore, in addition to conventional NPV analysis, for the first time, the finance team has also used the Black-Scholes option pricing (BSOP) model to help assess the value of the projects, incorporating options to delay decisions when making the investment decisions.

Further details of the projects, with the exception of Project E, are given in the table below.

Project	А	В	С	D	Е
Investment required	\$300m	\$300m	\$300m	\$300m	
Cash inflows (discounted to					
start of investment period)	\$350m	\$260m	\$330m	\$280m	See
Time	0	0	1	0.2	
Project net present value	\$50m	\$(40)m	\$30m	\$(20)m	below
Project value with an option to	No delay	No delay			
delay the investment decision	option	option	\$48·5m	\$9·62m	

The time shown in the table above refers to the number of years available to the company before it needs to decide on whether to undertake the project or not.

Project E

Project E is different to the other four projects. Whereas the other projects require an initial investment cash outflow, which is then followed by a number of positive annual cash inflows, Project E is a two-phase project. Project E requires an initial cash outflow immediately for phase one of the project and then a second cash outflow at the start of Year 5 for the phase two of the project. Ruva Co is not obliged to undertake phase two of the project and can stop further investment in Project E after phase one of the project is completed, if it wishes to do so. Phase one of Project E will last for four years and phase two of Project E will last for a further 10 years.

Project E, phase one

Phase one of the project will require an initial investment of \$190 million payable immediately, of which \$70 million relate to land and buildings and the balance remaining is invested in machinery. The machinery is eligible for annual tax allowances of 25% on a reducing balance basis. It is assumed that the machinery will have a zero value at the end of phase one. It is anticipated that when phase one ends in four years' time, the land and buildings will be sold but will have a value of zero due to the costs related to the sale.

Phase one of the project will also need working capital of \$30 million immediately. This is expected to increase by 20% every year and will be fully released back to Ruva Co on completion of that phase of the project.

For phase one, sales revenue in Year 1 is expected to be \$80 million. This will increase by 80% in Year 2 and then by the general rate of inflation in each of Years 3 and 4. Costs are expected to be \$50 million in Year 1 and then increase by 10% annually for the next three years.

Ruva Co pays corporation tax at an annual rate of 20% and tax is payable in the year the tax liability arises.

Project E, phase two

Phase two of Project E will require an investment cost of \$320 million payable at the start of Year 5. The annual inflation adjusted after-tax cash inflows for phase two are expected to be \$50 million. Ruva Co expects that phase two will be sold at the end of its ten-year life for \$60 million. There is considerable uncertainty surrounding these cash flows and their volatility is estimated at a standard deviation of 30%.

Other financial information

Ruva Co's market value of debt to market value of equity ratio is 40:60 and its annual cost of debt is 5%. Project E is a departure from Ruva Co's normal business activity and the all-equity financed cost of equity of a company engaged in the same line of business as Project E is 12%. Ruva Co's current cost of capital is estimated at 9%.

The annual rate of inflation in the country where Ruva Co is based is 4%, and the government's annual borrowing base rate is 3%.

The chief financial officer believes that phase two's initial investment cost of \$320 million at the start of Year 5 should be discounted at Ruva Co's cost of debt, as this would be a better estimate of its opportunity cost.

Ruva Co: project finance strategy

In addition to projects with a negative NPV being rejected, Ruva Co has also rejected some projects generating positive net present values. Ruva Co restricts finance for new projects through the employment of soft capital rationing. Therefore, finance is provided for projects on the basis that the positive net present value for every \$1 of capital invested is maximised.

The board of directors (BoD) has concluded that Ruva Co is at its optimum capital structure. The BoD is reluctant to move away from this optimum. Ruva Co also aims to pay dividends which grow at a steady rate annually. Additional finance for new projects is only made available from cash flows generated from ongoing projects and from any additional borrowing, such that the optimum capital structure is maintained after dividends have been paid.

Required:

- (a) With reference to Project E, discuss how real options build on net present value when evaluating investment decisions. (5 marks)
- (b) Prepare a report for the board of directors (BoD) of Ruva Co which:
 - (i) Estimates the value of Project E, assuming that Ruva Co does not delay the decision of whether to undertake phase two, but makes the decision at the commencement of phase one of the project;

(12 marks)

- (ii) Estimates the value of Project E, assuming that Ruva Co delays making the decision on phase two of the project; (8 marks)
- (iii) Discusses the reason(s) for the chief financial officer's belief in using 5% to discount the \$320 million cost of phase two of Project E and the assumptions made when estimating the value of that phase in part (b)(ii) above; (7 marks)
- (iv) Discusses the investment decision the BoD should make for each of the five projects: Project A to Project E. (6 marks)

Professional marks will be awarded in part (b) for the format, structure and presentation of the report.

(4 marks)

(c) Critically discuss the consequences of Ruva Co's employment of soft capital rationing when making project finance decisions. If Ruva Co decides not to employ soft capital rationing, advise where it could obtain the extra finance for its value added projects. (8 marks)

(50 marks)

Section B – TWO questions ONLY to be attempted

2 Clio

The Clio chain (Clio) of bookshops was founded ten years ago when its two founders opened four bookshops in towns in the country of Melpomene. The chain has expanded steadily over the last decade, mainly by opening new shops or buying existing single bookshops in towns in Melpomene. The one exception was the purchase of the five bookshops of the Erato chain two years ago. Currently Clio does not have any shops in the capital city or the three other largest cities in Melpomene.

Although Clio sells books online, its main focus has been sales through its stores. Its results have been significantly better than other bookshop chains in Melpomene, which have been adversely affected by customers buying more books online. Clio has had a policy of employing experienced retail staff and has encouraged them to acquire a wide range of knowledge about current books so that they can provide an informed service to customers. Clio has also tried to be a focus for communities. Every shop has a café, and book groups and other organisations are encouraged to meet in the shops. The shops enjoy a lot of autonomy if they are performing well.

65% of Clio's equity shares are currently owned by its two founders. 25% of equity shares are owned by Calliope, a venture capitalist, which invested in Clio when it purchased the Erato chain. Calliope would like Clio to be listed on Melpomene's stock exchange sometime in the next five years. At present Clio's board feels that all the shops in the chain are performing well, but that the chain needs to expand further in order for a listing to be successful.

Polymnia

Polymnia Books (Polymnia) was founded 120 years ago by Edward Euterpe. It has been owned and managed by members of the Euterpe family throughout its history, and is not listed on Melpomene's stock exchange. Polymnia is the second largest bookshop in Melpomene's capital city. Unlike other very large bookshops in the capital, Polymnia is not part of a chain.

Polymnia is located on a site, which it owns outright, in the middle of one of the busiest shopping streets in the capital of Melpomene. Polymnia has a large selection of books on a very wide range of subjects and also has music and second-hand book departments. It does not have a café or other facilities for customers. Polymnia does not currently make any sales online.

Polymnia's most recent financial results appear to be satisfactory. It has a lot of loyal customers and is a popular destination for tourists. Performance of major retailers based in the capital city has also been strong recently, a trend which is likely to continue for the next few years.

However, a number of criticisms have been made about Polymnia. Some customers find the store's layout confusing, it can be difficult to purchase books quickly and inventory turnover appears to be low. Polymnia has also been criticised for the lack of knowledge and unhelpfulness of its staff. Polymnia employs a few experienced employees in specialist areas, but to keep labour costs down it mostly employs staff aged under 30, who often leave within a year or two.

Polymnia's managing director for the last 45 years has recently died. Polymnia's shareholders will be willing to sell the bookshop if a satisfactory offer is made.

Purchase of Polymnia

The board of Clio has discussed purchasing Polymnia and believe that an offer of around \$20 million would be reasonable. However, initial contacts with Polymnia's board have indicated that the Euterpe family may be looking for an offer of around \$35 million.

The finance directors (FD) of Polymnia and Clio have been in contact. Polymnia's FD has stated its forecast free cash flows to the firm in the next year will be \$1,350,000. Polymnia's FD expects the annual free cash flows to rise by the following percentages in subsequent years:

Year	2	3	4	5 onwards
	15%	10%	10%	5%

Clio's FD, however, believes that it would be better to assume that annual free cash flows will remain unchanged after Year 4. He also believes that Clio will need to spend \$700,000 in Year 1 and \$500,000 in Year 2 on non-current assets for Polymnia, in addition to the non-current asset expenditure already included in the free cash flows. However, Clio's FD expects the acquisition to generate pre-tax synergies of \$400,000 each year for the next four years for Clio. Polymnia has no long-term debt and its beta factor can be assumed to be 0.8, which Clio FD's will also assume to be the ungeared beta of Clio. If the purchase cost is around \$20 million, the purchase will be mainly funded by debt, and Clio's debt:equity ratio will be 45:55 as a result. Clio's current pre-tax cost of debt is $7\cdot3\%$. The current corporation tax rate in Melpomene is 25%. The annual risk-free rate of return is $3\cdot5\%$ and the market risk premium is $6\cdot5\%$.

Clio's FD wishes to prepare a valuation of Polymnia on an assets basis as an alternative to a cash flow-based valuation. One of Clio's other directors has wondered whether the goodwill which many of Polymnia's customers have towards it should be factored in as an asset. Another director is sceptical about basing a valuation on cash flow forecasts provided by Polymnia and suggests that a valuation based on Polymnia's current earnings may be more realistic.

Required:

- (a) Discuss the advantages of, and problems with, Clio purchasing Polymnia. (9 marks)
- (b) Prepare valuations of Polymnia based on forecast free cash flows to the firm method and assuming (i) 5% growth in cash flows after Year 4 and (ii) zero growth in cash flows after Year 4. Discuss your results and the assumptions made in the light of the bid values planned by Clio and required by Polymnia's shareholders.

(10 marks)

(c) Discuss the issues involved with using asset or price/earnings-based valuations of Polymnia as the basis for determining what the offer should be. (6 marks)

(25 marks)

3 Arthuro Co group

Arthuro Co is based in Hittyland and is listed on Hittyland's stock exchange. Arthuro Co has one wholly-owned subsidiary, Bowerscots Co, based in the neighbouring country of Owlia. Hittyland and Owlia are in a currency union and the currency of both countries is the \$.

Arthuro Co purchased 100% of Bowerscots Co's share capital three years ago. Arthuro Co has the power under the acquisition to determine the level of dividend paid by Bowerscots Co. However, Arthuro Co's board decided to let Bowerscots Co's management team have some discretion when making investment decisions. Arthuro Co's board decided that it should receive dividends of 60% of Bowerscots Co's post-tax profits and has allowed Bowerscots Co to use its remaining retained earnings to fund investments chosen by its management. A bonus linked to Bowerscots Co's after-tax profits is a significant element of Bowerscots Co's managers' remuneration.

Bowerscots Co operates in a very competitive environment. Recently, a senior member of its management team has left to join a competitor.

Arthuro Co's dividend policy

Until three months ago, Arthuro Co had 90 million \$2 equity shares in issue and \$135 million 8% bonds. Three months ago it made a 1 for 3 rights issue. A number of shareholders did not take up their rights, but sold them on, so there have been changes in its shareholder base. Some shareholders expressed concern about dilution of their dividend income as a result of the rights issue. Therefore, Arthuro Co's board felt it had to promise, for the foreseeable future, at least to maintain the dividend of \$0.74 per equity share, which it paid for the two years before the rights issue.

Arthuro Co's board is nevertheless concerned about whether it will have sufficient funds available to fulfil its promise about the dividend. It has asked the finance director to forecast its dividend capacity based on assumptions about what will happen in a 'normal' year. The finance director has made the following assumptions in the forecast:

- 1. Sales revenue can be assumed to be 4% greater than the most recent year's of \$520 million.
- 2. The operating profit margin can be assumed to be 20%.
- 3. Operating profit can be assumed to be reported after charging depreciation of \$30 million and profit on disposal of non-current assets of \$5.9 million. The cost of the non-current assets sold can be assumed to be \$35 million and its accumulated depreciation to be \$24.6 million. Depreciation is allowable for tax and the profit on disposal is fully chargeable to tax.
- 4. The net book value of non-current assets at the year end in the most recent accounts was \$110 million. To maintain productive capacity, sufficient investment to increase this net book value figure 12 months later by 4% should be assumed, in line with the increase in sales. The calculation of investment required for the year should take into account the depreciation charged of \$30 million, and net book value of the non-current assets disposed of during the year.
- 5. A \$0.15 investment in working capital can be assumed for every \$1 increase in sales revenue.
- 6. Bowerscots Co's pre-tax profits can be assumed to be \$45 million.

Arthuro Co's directors have decided that if there is a shortfall of dividend capacity, compared with the dividends required to maintain the current dividend level, the percentage of post-tax profits of Bowerscots Co paid as dividend should increase, if necessary up to 100%.

Taxation

Arthuro Co pays corporation tax at 30% and Bowerscots Co pays corporation tax at 20%. A withholding tax of 5% is deducted from any dividends remitted by Bowerscots Co. There is a bilateral tax treaty between Hittyland and Owlia. Corporation tax is payable by Arthuro Co on profits declared by Bowerscots Co, but Hittyland gives full credit for corporation tax already paid in Owlia. Hittyland gives no credit for withholding tax paid on dividends in Owlia.

Required:

- (a) (i) Estimate Arthuro Co's forecast dividend capacity for a 'normal' year; (11 marks)
 - (ii) Estimate the level of dividend required from Bowerscots Co to give Arthuro Co sufficient dividend capacity to maintain its dividend level of \$0.74 per equity share. (3 marks)

(b) Arthuro Co has decided to increase its level of dividend from Bowerscots Co if its dividend capacity is insufficient.

Required:

(ii) Discuss the agency problems, and how they might be resolved, with this decision. (6 marks)

(25 marks)

(5 marks)

4 Hickamore Co is a large company based in Sohland, where the currency is the dollar. Assume it is now 1 June 2018.

New site

Hickamore Co is planning to purchase a new site for its operations in Sohland's South province and fund the purchase from sales of two of its current sites, in Sohland's North and Central provinces. Hickamore Co's board expects that North province site will have been sold by the time it completes the purchase for the new site. It anticipates completing the purchase and being liable to pay the purchase price on 1 December 2018.

Hickamore Co's board's current best guess is that the sale of the Central province site will be completed on 1 July 2019. The board therefore anticipates needing a bridging loan of \$10.5 million for the seven-month period beginning on 1 December 2018.

Hedging interest payment

Interest rates in Sohland have recently been volatile. Hickamore Co's finance director is considering whether to hedge the interest payment on the bridging loan which the company needs. Hickamore Co is able to borrow at the National Bank base rate plus 40 basis points. The National Bank base rate is currently 4.4%. Hickamore Co's finance director predicts that political pressures will mean that the base rate will not increase above 5% and could fall as low as 3.4% if the Sohland government wishes to gain popularity by cutting interest rates.

Hickamore Co's finance director wishes to determine what would be the results of using exchange-traded derivatives to hedge the interest due if interest rates were at 5% or 3.4%, which he regards as the extreme values. He has obtained the following information about exchange-traded \$ futures and options:

Three-month December \$ futures, \$500,000 contract size

Prices are quoted in basis points at 100 – annual % yield: 94.83

Options on three-month December \$ futures, \$500,000 contract size, option premiums are in annual %

Calls	Strike price	Puts
0.614	94.75	0.108
0.129	95.75	0.584

It can be assumed that futures and options contracts are settled at the end of each month. Basis can be assumed to diminish to zero at contract maturity at a constant rate, based on monthly time intervals. It can also be assumed that there is no basis risk and there are no margin requirements.

Required:

- (a) Recommend a hedging strategy for the bridging loan, based on the hedging choices which have been specified, if interest rates increase by 0.6% or decrease by 1.0%. Support your answer with appropriate calculations and discussion. (17 marks)
- (b) Explain how the uncertainty in volatility is measured in option pricing theory and discuss the influences on the level of uncertainty. (3 marks)

Hickamore Co is also looking at its longer-term financing strategy for the funding of future investments. In the past, Hickamore Co has mostly used floating rate finance. However, its finance director is considering whether the company should enter a swap for fixed rate borrowing in the future, if interest rates continue to be volatile. If Hickamore Co uses a swap on planned borrowing in the future, the finance director wants to ensure that the fixed rate it is offered is capped at a maximum level.

Required:

(c) Explain what is meant by a swaption and use a 1 x 4 swaption at an interest rate of 5% to demonstrate how a swaption might fulfil the requirements of Hickamore Co's finance director. (5 marks)

(25 marks)

Formulae

Modigliani and Miller Proposition 2 (with tax)

$$k_{e} = k_{e}^{i} + (1 - T)(k_{e}^{i} - k_{d})\frac{V_{d}}{V_{e}}$$

The Capital Asset Pricing Model

$$\mathsf{E}(\mathsf{r}_{\mathsf{i}}) = \mathsf{R}_{\mathsf{f}} + \beta_{\mathsf{i}}(\mathsf{E}(\mathsf{r}_{\mathsf{m}}) - \mathsf{R}_{\mathsf{f}})$$

The asset beta formula

$$\boldsymbol{\beta}_{a} = \left[\frac{V_{e}}{(V_{e} + V_{d}(1 - T))}\boldsymbol{\beta}_{e}\right] + \left[\frac{V_{d}(1 - T)}{(V_{e} + V_{d}(1 - T))}\boldsymbol{\beta}_{d}\right]$$

The Growth Model

$$P_{o} = \frac{D_{o}(1+g)}{(r_{e} - g)}$$

Gordon's growth approximation

 $g = br_e$

The weighted average cost of capital

$$WACC = \left[\frac{V_e}{V_e + V_d}\right]k_e + \left[\frac{V_d}{V_e + V_d}\right]k_d(1 - T)$$

The Fisher formula

$$(1+i) = (1+r)(1+h)$$

Purchasing power parity and interest rate parity

$$S_1 = S_0 x \frac{(1+h_c)}{(1+h_b)}$$
 $F_0 = S_0 x \frac{(1+i_c)}{(1+i_b)}$

Modified Internal Rate of Return

$$MIRR = \left[\frac{PV_R}{PV_I}\right]^{\frac{1}{n}} \left(1 + r_e\right) - 1$$

The Black-Scholes option pricing model

$$c = P_a N(d_1) - P_e N(d_2) e^{-rt}$$

Where:

$$d_{1} = \frac{\ln(P_{a} / P_{e}) + (r + 0.5s^{2})t}{s\sqrt{t}}$$
$$d_{2} = d_{1} - s\sqrt{t}$$

The Put Call Parity relationship

$$p = c - P_a + P_e e^{-rt}$$

Present Value Table

Present value of 1 i.e. $(1 + r)^{-n}$

Where r = discount rate

n = number of periods until payment

					2.0000						
Period (n)	ls 1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909	1
2	0.980	0.961	0.943	0.925	0.907	0.890	0.873	0.857	0.842	0.826	2
3	0.971	0.942	0.915	0.889	0.864	0.840	0.816	0.794	0.772	0.751	3
4	0.961	0.924	0.888	0.855	0.823	0.792	0.763	0.735	0.708	0.683	4
5	0.951	0.906	0.863	0.822	0.784	0.747	0.713	0.681	0.650	0.621	5
6	0.942	0.888	0.837	0.790	0.746	0.705	0.666	0.630	0.596	0.564	6
7	0.933	0.871	0.813	0.760	0.711	0.665	0.623	0.583	0.547	0.513	7
8	0.923	0.853	0.789	0.731	0.677	0.627	0.582	0.540	0.502	0.467	8
9	0.914	0.837	0.766	0.703	0.645	0.592	0.544	0.200	0.460	0.424	9
10	0.905	0.820	0.744	0.676	0.614	0.558	0.208	0.463	0.422	0.386	10
11	0.896	0.804	0.722	0.650	0.585	0.527	0.475	0.429	0.388	0.350	11
12	0.887	0.788	0.701	0.625	0.557	0.497	0.444	0.397	0.356	0.319	12
13	0.879	0.773	0.681	0.601	0.530	0.469	0.415	0.368	0.326	0.290	13
14	0.870	0.758	0.661	0.577	0.505	0.442	0.388	0.340	0.299	0.263	14
15	0.861	0.743	0.642	0.555	0.481	0.417	0.362	0.315	0.275	0.239	15
(n)	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	
1	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833	1
2	0.812	0.797	0.783	0.769	0.756	0.743	0.731	0.718	0.706	0.694	2
3	0.731	0.712	0.693	0.675	0.658	0.641	0.624	0.609	0.593	0.579	3
4	0.659	0.636	0.613	0.592	0.572	0.552	0.534	0.516	0.499	0.482	4
5	0.593	0.567	0.543	0.519	0.497	0.476	0.456	0.437	0.419	0.402	5
6	0.535	0.507	0.480	0.456	0.432	0·410	0.390	0.370	0.352	0.335	6
7	0.482	0.452	0.425	0.400	0.376	0.354	0.333	0.314	0.296	0.279	7
8	0.434	0.404	0.376	0.351	0.327	0.305	0.285	0.266	0.249	0.233	8
9	0.391	0.361	0.333	0.308	0.284	0.263	0.243	0.225	0.209	0.194	9
10	0.352	0.322	0.295	0.270	0.247	0.227	0.208	0.191	0.176	0.162	10
11	0.317	0.287	0·261	0.237	0·215	0·195	0·178	0.162	0·148	0.135	11
12	0.286	0.257	0.231	0.208	0.187	0.168	0.152	0.137	0.124	0.112	12
13	0.258	0.229	0.204	0.182	0.163	0.145	0.130	0.116	0.104	0.093	13
14	0.232	0.205	0.181	0.160	0.141	0.125	0.111	0.099	0.088	0.078	14
15	0.209	0.183	0.160	0.140	0.123	0.108	0.095	0.084	0.074	0.065	15

Annuity Table

Present value of an annuity of 1 i.e. $\frac{1 - (1 + r)^{-n}}{r}$

Where r = discount raten = number of periods

Discount rate (r)

Periods (n)	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%	
1 2	0·990 1·970	0·980 1·942	0·971 1·913	0·962 1·886	0·952 1·859	0·943 1·833	0·935 1·808	0·926 1·783	0·917 1·759	0·909 1·736	1 2
3	2·941	2.884	2·829	2·775	2·723	2·673	1 000 2·624	2·577	2·531	2·487	3
4	3.902	3.808	3.717	3.630	3.546	3.465	3.387	3.312	3.240	3.170	4
5	4.853	4.713	4.580	4.452	4.329	4·212	4.100	3.993	3.890	3.791	5
6	5.795	5.601	5·417	5.242	5.076	4·917	4.767	4.623	4.486	4·355	6
7	6.728	6.472	6.230	6.002	5.786	5.582	5.389	5.206	5.033	4.868	7
8	7.652	7.325	7.020	6.733	6.463	6.210	5.971	5.747	5.535	5.335	8
9	8.566	8.162	7.786	7.435	7.108	6.802	6.515	6.247	5.995	5.759	9
10	9.471	8.983	8.530	8.111	7.722	7.360	7.024	6.710	6.418	6.145	10
11	10.368	9.787	9.253	8·760	8.306	7.887	7.499	7.139	6.805	6.495	11
12	11.255	10.575	9.954	9.385	8.863	8·384	7.943	7.536	7.161	6·814	12
13	12.134	11.348	10.635	9.986	9.394	8.853	8.358	7.904	7.487	7.103	13
14	13.004	12.106	11.296	10.563	9.899	9.295	8·745	8·244	7.786	7.367	14
15	13.865	12.849	11.938	11.118	10.380	9.712	9.108	8.559	8.061	7.606	15
(n)	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%	
1	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833	1
1 2	0·901 1·713	0·893 1·690	0·885 1·668	0·877 1·647	0·870 1·626	0·862 1·605	0·855 1·585	0·847 1·566	0·840 1·547	0·833 1·528	2
1 2 3	0·901 1·713 2·444	0·893 1·690 2·402	0·885 1·668 2·361	0·877 1·647 2·322	0·870 1·626 2·283	0·862 1·605 2·246	0·855 1·585 2·210	0·847 1·566 2·174	0·840 1·547 2·140	0·833 1·528 2·106	2 3
1 2 3 4	0·901 1·713 2·444 3·102	0·893 1·690 2·402 3·037	0·885 1·668 2·361 2·974	0·877 1·647 2·322 2·914	0·870 1·626 2·283 2·855	0·862 1·605 2·246 2·798	0·855 1·585 2·210 2·743	0·847 1·566 2·174 2·690	0·840 1·547 2·140 2·639	0·833 1·528 2·106 2·589	2 3 4
1 2 3	0·901 1·713 2·444	0·893 1·690 2·402	0·885 1·668 2·361	0·877 1·647 2·322	0·870 1·626 2·283	0·862 1·605 2·246	0·855 1·585 2·210	0·847 1·566 2·174	0·840 1·547 2·140	0·833 1·528 2·106	2 3
1 2 3 4 5 6	0.901 1.713 2.444 3.102 3.696 4.231	0.893 1.690 2.402 3.037 3.605 4.111	0.885 1.668 2.361 2.974 3.517 3.998	0.877 1.647 2.322 2.914 3.433 3.889	0.870 1.626 2.283 2.855 3.352 3.784	0.862 1.605 2.246 2.798 3.274 3.685	0.855 1.585 2.210 2.743 3.199 3.589	0.847 1.566 2.174 2.690 3.127 3.498	0.840 1.547 2.140 2.639 3.058 3.410	0.833 1.528 2.106 2.589 2.991 3.326	2 3 4 5 6
1 2 3 4 5 6 7	0.901 1.713 2.444 3.102 3.696 4.231 4.712	0.893 1.690 2.402 3.037 3.605 4.111 4.564	0.885 1.668 2.361 2.974 3.517 3.998 4.423	0.877 1.647 2.322 2.914 3.433 3.889 4.288	0.870 1.626 2.283 2.855 3.352 3.784 4.160	0.862 1.605 2.246 2.798 3.274 3.685 4.039	0.855 1.585 2.210 2.743 3.199 3.589 3.922	0.847 1.566 2.174 2.690 3.127 3.498 3.812	0.840 1.547 2.140 2.639 3.058 3.410 3.706	0.833 1.528 2.106 2.589 2.991 3.326 3.605	2 3 4 5 6 7
1 2 3 4 5 6 7 8	0.901 1.713 2.444 3.102 3.696 4.231 4.712 5.146	0.893 1.690 2.402 3.037 3.605 4.111 4.564 4.968	0.885 1.668 2.361 2.974 3.517 3.998 4.423 4.799	0.877 1.647 2.322 2.914 3.433 3.889 4.288 4.639	0.870 1.626 2.283 2.855 3.352 3.784 4.160 4.487	0.862 1.605 2.246 2.798 3.274 3.685 4.039 4.344	0.855 1.585 2.210 2.743 3.199 3.589 3.922 4.207	0.847 1.566 2.174 2.690 3.127 3.498 3.812 4.078	0.840 1.547 2.140 2.639 3.058 3.410 3.706 3.954	0.833 1.528 2.106 2.589 2.991 3.326 3.605 3.837	2 3 4 5 6 7 8
1 2 3 4 5 6 7 8 9	0.901 1.713 2.444 3.102 3.696 4.231 4.712 5.146 5.537	0.893 1.690 2.402 3.037 3.605 4.111 4.564 4.968 5.328	0.885 1.668 2.361 2.974 3.517 3.998 4.423 4.799 5.132	0.877 1.647 2.322 2.914 3.433 3.889 4.288 4.639 4.946	0.870 1.626 2.283 2.855 3.352 3.784 4.160 4.487 4.772	0.862 1.605 2.246 2.798 3.274 3.685 4.039 4.344 4.607	0.855 1.585 2.210 2.743 3.199 3.589 3.922 4.207 4.451	0.847 1.566 2.174 2.690 3.127 3.498 3.812 4.078 4.303	0.840 1.547 2.140 2.639 3.058 3.410 3.706 3.954 4.163	0.833 1.528 2.106 2.589 2.991 3.326 3.605 3.837 4.031	2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8	0.901 1.713 2.444 3.102 3.696 4.231 4.712 5.146	0.893 1.690 2.402 3.037 3.605 4.111 4.564 4.968	0.885 1.668 2.361 2.974 3.517 3.998 4.423 4.799	0.877 1.647 2.322 2.914 3.433 3.889 4.288 4.639	0.870 1.626 2.283 2.855 3.352 3.784 4.160 4.487	0.862 1.605 2.246 2.798 3.274 3.685 4.039 4.344	0.855 1.585 2.210 2.743 3.199 3.589 3.922 4.207	0.847 1.566 2.174 2.690 3.127 3.498 3.812 4.078	0.840 1.547 2.140 2.639 3.058 3.410 3.706 3.954	0.833 1.528 2.106 2.589 2.991 3.326 3.605 3.837	2 3 4 5 6 7 8
1 2 3 4 5 6 7 8 9	0.901 1.713 2.444 3.102 3.696 4.231 4.712 5.146 5.537	0.893 1.690 2.402 3.037 3.605 4.111 4.564 4.968 5.328	0.885 1.668 2.361 2.974 3.517 3.998 4.423 4.799 5.132	0.877 1.647 2.322 2.914 3.433 3.889 4.288 4.639 4.946	0.870 1.626 2.283 2.855 3.352 3.784 4.160 4.487 4.772	0.862 1.605 2.246 2.798 3.274 3.685 4.039 4.344 4.607	0.855 1.585 2.210 2.743 3.199 3.589 3.922 4.207 4.451	0.847 1.566 2.174 2.690 3.127 3.498 3.812 4.078 4.303	0.840 1.547 2.140 2.639 3.058 3.410 3.706 3.954 4.163	0.833 1.528 2.106 2.589 2.991 3.326 3.605 3.837 4.031	2 3 4 5 6 7 8 9
1 2 3 4 5 6 7 8 9 10	0.901 1.713 2.444 3.102 3.696 4.231 4.712 5.146 5.537 5.889	0.893 1.690 2.402 3.037 3.605 4.111 4.564 4.968 5.328 5.650	0.885 1.668 2.361 2.974 3.517 3.998 4.423 4.799 5.132 5.426	0.877 1.647 2.322 2.914 3.433 3.889 4.288 4.639 4.946 5.216	0.870 1.626 2.283 2.855 3.352 3.784 4.160 4.487 4.772 5.019	0.862 1.605 2.246 2.798 3.274 3.685 4.039 4.344 4.607 4.833	0.855 1.585 2.210 2.743 3.199 3.589 3.922 4.207 4.451 4.659	0.847 1.566 2.174 2.690 3.127 3.498 3.812 4.078 4.303 4.494	0.840 1.547 2.140 2.639 3.058 3.410 3.706 3.954 4.163 4.339	0.833 1.528 2.106 2.589 2.991 3.326 3.605 3.837 4.031 4.192	2 3 4 5 6 7 8 9 10
1 2 3 4 5 6 7 8 9 10 11	0.901 1.713 2.444 3.102 3.696 4.231 4.712 5.146 5.537 5.889 6.207	0.893 1.690 2.402 3.037 3.605 4.111 4.564 4.968 5.328 5.650 5.938	0.885 1.668 2.361 2.974 3.517 3.998 4.423 4.799 5.132 5.426 5.687	0.877 1.647 2.322 2.914 3.433 3.889 4.288 4.639 4.946 5.216 5.453	0.870 1.626 2.283 2.855 3.352 3.784 4.160 4.487 4.772 5.019 5.234	0.862 1.605 2.246 2.798 3.274 3.685 4.039 4.344 4.607 4.833 5.029	0.855 1.585 2.210 2.743 3.199 3.589 3.922 4.207 4.451 4.659 4.836	0.847 1.566 2.174 2.690 3.127 3.498 3.812 4.078 4.303 4.494 4.656	0.840 1.547 2.140 2.639 3.058 3.410 3.706 3.954 4.163 4.339 4.486	0.833 1.528 2.106 2.589 2.991 3.326 3.605 3.837 4.031 4.192 4.327	2 3 4 5 6 7 8 9 10 11
1 2 3 4 5 6 7 8 9 10 11 12	0.901 1.713 2.444 3.102 3.696 4.231 4.712 5.146 5.537 5.889 6.207 6.492	0.893 1.690 2.402 3.037 3.605 4.111 4.564 4.968 5.328 5.650 5.938 6.194	0.885 1.668 2.361 2.974 3.517 3.998 4.423 4.799 5.132 5.426 5.687 5.918	0.877 1.647 2.322 2.914 3.433 3.889 4.288 4.639 4.946 5.216 5.453 5.660	0.870 1.626 2.283 2.855 3.352 3.784 4.160 4.487 4.772 5.019 5.234 5.421	0.862 1.605 2.246 2.798 3.274 3.685 4.039 4.344 4.607 4.833 5.029 5.197	0.855 1.585 2.210 2.743 3.199 3.589 3.922 4.207 4.451 4.659 4.836 4.988	0.847 1.566 2.174 2.690 3.127 3.498 3.812 4.078 4.303 4.494 4.656 4.793	0.840 1.547 2.140 2.639 3.058 3.410 3.706 3.954 4.163 4.339 4.486 4.611	0.833 1.528 2.106 2.589 2.991 3.326 3.605 3.837 4.031 4.192 4.327 4.439	2 3 4 5 6 7 8 9 10 11 12
1 2 3 4 5 6 7 8 9 10 11 12 13	0.901 1.713 2.444 3.102 3.696 4.231 4.712 5.146 5.537 5.889 6.207 6.492 6.750	0.893 1.690 2.402 3.037 3.605 4.111 4.564 4.968 5.328 5.650 5.938 6.194 6.424	0.885 1.668 2.361 2.974 3.517 3.998 4.423 4.799 5.132 5.426 5.687 5.918 6.122	0.877 1.647 2.322 2.914 3.433 3.889 4.288 4.639 4.946 5.216 5.453 5.660 5.842	0.870 1.626 2.283 2.855 3.352 3.784 4.160 4.487 4.772 5.019 5.234 5.421 5.583	0.862 1.605 2.246 2.798 3.274 3.685 4.039 4.344 4.607 4.833 5.029 5.197 5.342	0.855 1.585 2.210 2.743 3.199 3.589 3.922 4.207 4.451 4.659 4.836 4.988 5.118	0.847 1.566 2.174 2.690 3.127 3.498 3.812 4.078 4.303 4.494 4.656 4.793 4.910	0.840 1.547 2.140 2.639 3.058 3.410 3.706 3.954 4.163 4.339 4.486 4.611 4.715	0.833 1.528 2.106 2.589 2.991 3.326 3.605 3.837 4.031 4.192 4.327 4.439 4.533	2 3 4 5 6 7 8 9 10 11 12 13

Standard normal distribution table

	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0·1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1808	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0·2611	0.2642	0.2673	0.2704	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0·3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3208	0.3531	0.3554	0.3577	0.3599	0.3621
$1 \cdot 1$	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4222	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4756	0.4761	0.4767
2.0	0.4772	0.4778	0.4783	0.4788	0.4793	0.4798	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
	0 4020	0 4040	0 40 4 1	0 10 12	0 1015	0 4040	0 40 40	0 10 10	0 4051	0 4050
2·5	0·4938	0·4940	0·4941	0·4943	0·4945	0.4946	0·4948	0·4949	0·4951	0.4952
2·6	0·4953	0.4955	0·4956	0·4957	0·4959	0·4960	0·4961	0·4962	0.4963	0·4964
2·7	0·4965	0·4966	0·4967	0·4968	0·4969	0·4970	0·4971	0·4972	0.4973	0·4974
2·8	0·4974	0.4975	0·4976	0·4977	0·4977	0.4978	0·4979	0·4979	0·4980	0·4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
2.0	0.4987	0.4007	0.4007	0.4000	0.4000	0.4090	0.4090	0.4090	0.4990	0.4000
3.0	0.4301	0.4987	0.4987	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990	0.4990

This table can be used to calculate N(d), the cumulative normal distribution functions needed for the Black-Scholes model of option pricing. If $d_i > 0$, add 0.5 to the relevant number above. If $d_i < 0$, subtract the relevant number above from 0.5.

End of Question Paper